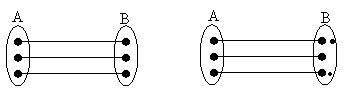
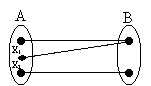
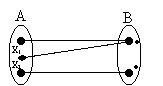
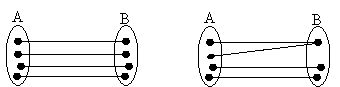
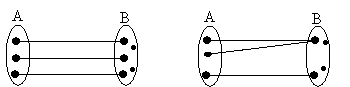
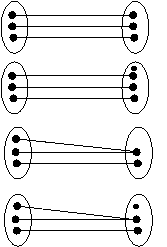
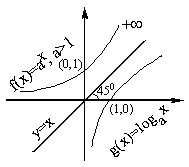
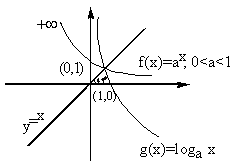
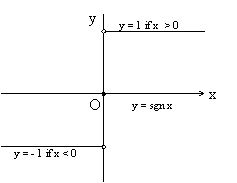
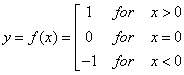
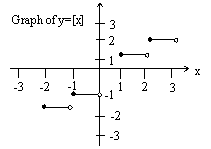
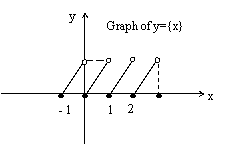
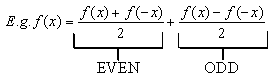
1. **General Definition :** If to every value (Considered as real unless other-wise stated) of a variable x, which belongs to some collection (Set) A, there corresponds one and only one finite value of the quantity y, then y is said to be a function (Single valued) of x or a dependent variable defined on the set A; x is the argument or independent variable.  
  
If to every value of x belonging to some set A there corresponds one or several values of the variable y, then y is called a multiple valued function of x defined on A. Conventionally the word ''**Function**'' is used only as the meaning of a single valued function, if not otherwise stated. Pictorially: http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002.gify is called the image of x and x is the pre-image of y under f.  
Every function from http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004.gifsatisfies the following conditions.  
a) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006.gif  
b) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image008.gifand   
c) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010.gif  
  
2. **Domain, Co-Domain & Range Of a Function :** Let http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image012.gif, then the set A is known as the domain of f and the set B is known as co-domain of f. The set of all ''f'' images of elements of A is known as the range of f. Thus :   
Domain of http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014.gif  
Range of http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image016.gif  
  
It should be noted that range is a subset of co-domain. Sometimes if only f(x) is given then domain is set of those values of ''x'' for which f(x) exists or is defined.  
To find the range of a function, there is n''t any particular approach, but student will find one of these approaches useful.  
i) When a function is given in the form y = f(x), express if possible ''x'' as a function of ''y'' i.e. x = g(y). Find the domain of ''g''. This will become range of ''f''.  
ii) If y = f(x) is a continuous or piece-wise continuous function, then range of ''f'' will be union of   
[Minmf(x), Maxmf(x)] in all such intervals where f(x) is continuous/piece-wise continuous.  
  
3. **Classification of Functions :**   
**Functions can be classified into two categories :**   
i) **One-One Function (Injective mapping) or Many - One Function :** A function is said to be a one-one function or injective mapping if different elements of A have different f images in B. Thus for .  
  
**http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0000.gif**  
**Diagramatically an injective mapping can be shown as**   
  
  
OR  
**Note :** (a) Any function which is entirely increasing or decreasing in whole domain, then f(x) is one-one.   
(b) If any line parallel to x-axis cuts the graph of the function atmost at one point, then the function is one-one.   
**Many - One Function :** A function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0001.gifis said to be a many one function if two or more elements of A have the same f image in B. Thus http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0002.gifis many one if for :  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0000.gifbut http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0000.gif  
  
**Diagramatically a many one mapping can be shown as**  
  
OR   
  
**Note :** (a) Any continuous function which has atleast one local maximum or local minimum, then f(x) is many-one. In other words, if there is even a single line parallel to x- axis cuts he graph of the function atleast at two points, then f is many - one.   
(b) If a function is one-one, it cannot be many-one and vice versa.  
(c) All functions can be categorized as one-one or many-one  
  
**ii)** **Onto function (Surjective mapping) or into function :**   
If the function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010_0000.gifis such that each element in B (co-domain) must have atleast one pre-image in A, then we say that f is a function of A ''onto'' B. Thus http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010_0001.gifis surjective iff http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image013.gifsome http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image015.gifsuch that f(a) = b.  
**Diagramatically surjective mapping can be shown as**  
  
**OR**   
**Note that** : If range = Co-domain, then f(x) is onto.  
  
**Into Function :** If http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0003.gifis such that there exists atleast one element in co-domain which is not the image of any element in domain, then f(x) is into.  
**Diagramatically into function can be shown as**  
  
**OR**  
  
 **Note that :** If a function is onto, it cannot be into and vice versa.  
Thus a function can be one of these four types :  
a) one-one onto (injective and surjective)  
b) one-one into (injective and surjective)  
c) many - one onto (surjective but not injective)   
d) many-one into (neither surjective nor injective)  
( domain in each case is http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0001.gif)  
**Note :** a) If f is both injective and surjective, then it is called a **Bijective** mapping. The bijective functions are also named as invertible, non-singular or biuniform functions.   
b) If a set A contains n distinct elements then the number of different functions defined from http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image008_0001.gifis nnand out of it n! are one one.  
  
4. **Algebraic Operations On Functions :** If f& g are real valued functions of x with domain set A, B respectively, then both f and g are defined in http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0002.gif. Now we define f + g, f - g, (f.g) and (f/g) as follows:  
i) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010_0002.gif  
ii) (f.g) (x) = f(x). g(x)  
iii) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image012_0000.gifdomain is http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014_0000.gif  
  
5. **Important Types of Functions :**  
**i) Polynomial Function :** If a function f is defined by http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/1.gifwhere n is a non negative integer and a0, a1, a2,...........anare real numbers and http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image016_0001.gif, then f is called a polynomial function of degree n.  
**Note :** a) A polynomial of degree one with no constant term is called an odd linear function i.e. f(x) = ax, http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0004.gif  
b) There are two polynomial functions, satisfying the relation :   
f(x).f(1/x) = f(x) + f(1/x). They are :  
i) f(x) = xn+ 1 and ii) f(x) = 1 - xn, where n is a positive integer.  
  
**ii)** **Algebraic Function :** y is an algebraic function of x, if it is a function that satisfies an algebraic equation of the form,http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/2.gifWhere n is a positive integer and P0(x), P1(x).........are Polynomials in x.  
E.g. http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0002.gifis an algebraic function, since it satisfies the equation y2- x2= 0.   
**Note : that** all polynomial functions are Algebraic but not the converse. A function that is not algebraic is called **Transcedental Function**.  
**iii) Fractional Rational Function :** A rational function is a function of the form. http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0003.gif, where g(x) and h(x) are polynomials and h(x)   
**iv) Exponential Function :** A function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/3.gifis called an exponential function. The inverse of the exponential function is called the logarithmic function. i.e. g(x) = logax.   
**Note that** f(x) and g(x) are inverse of each other & their graphs are as shown.  
  
  
 **v) Absolute Value Function :** A function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image005.gifis called the absolute value function or Modulus function.  
It is defined as : http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image007_0000.gif  
  
**vi) Signum Function :**   
A function y = f(x) = Sgn (x) is defined as follows :   
  
It is also written as Sgn http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image011.gif:   
  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014_0001.gif  
  
**vii) Greatest Integer Or Step function :**   
The function y = f(x) = [x] is called the greatest   
integer function where [x] denotes the greatest   
integer less than or equal to x.   
**Note that** for :  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image016_0002.gif; [x] = -1  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image018.gif; [x] = 0  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image020.gif; [x] = 1  
http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image022.gif; [x] = 2 and so on  
  
**Properties of greatest integer function :**   
a) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image024.gifand http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image026.gif  
b) [x + m] = [x] + m if m is an integer.  
c) [x] + [y] [ x + y] [x] + [y] + 1  
  
d) [x] + [-x] = 0 if x is an integer  
= -1 otherwise  
  
**viii) Fractional Part Function :**   
It is defined as :  
g(x) = {x} = x - [x]   
e.g. the fractional part of the no.2.1 is 2.1 - 2 = 0.1   
and the fractional part of -3.7 is 0.3.   
The period of this function is 1 and graph  
of this function is as shown.  
  
**ix) Identity Function :** The function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0006.gifdefined by http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0003.gifis called the identity of A and is denoted by IA. It is easy to observe that identity function is a bijection.  
  
**x) Constant function :** A function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0004.gifis said to be a constant function if every element of A has the same f image in B. Thus http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image008_0003.gifis a constant function.   
**Note that** the range of a constant function is a singelton and a constant function may be one-one or many-one, onto.  
  
**6. Homogeneous Functions :** An integral function is said to be homogeneous with respect to any set of variables when each of its terms is of the same degree with respect to those variables. For example 5x2+ 3y2- xy is homogeneous in x and y  
  
**7. Bounded Function :** A function is said to be bounded if http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010_0003.gif, where M is a finite quantity  
  
**8. Implicit & Explicit Function :** A function defined by an equation not solved for the dependent variable is called an **Implicit Function** . For eg. the equation x3+ y3= 1 defines as an implicit function. If y has been expressed in terms of x alone then it is called an **Explicit Function**.  
  
**9. Odd & Even Functions :** If f(-x) = f(x) for all x in the domain of ''f'' then f is said to be an even function.   
E.g. f(x) = cos x ; g(x) = x2+ 3.   
If f(- x) = -f (x) for all x in the domain of ''f'' then f is said to be an odd function.  
E.g. f(x) = sin x ; g(x) = x3+ x.   
**Note** :   
(a) http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image012_0002.gifis even and http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014_0002.gifis odd  
(b) A function may neither be odd nor even.  
(c) Inverse of an even function is not defined.  
(d) Every even function is symmetric about the y-axis and every odd function is symmetric about the origin.  
(e) Every function can be expressed as the sum of an even and an odd function.  
  
(f) The only function which is defined on the entire number line and is even and odd at the same time is f(x) = 0.  
(g) If f and g both are even or both are odd then the function **f.g** will be even but if any one of them is odd then **f.g** will be odd.  
  
**10. Periodic Function :** A function f(x) is called periodic if there exists a +ve number T (T>0) called the period of the function such that f(x+ T) = f(x), for all values of x within the domain of x.  
E.g. The function sinx and cosx both are periodic over 2 http://testonline.in/templates/default/images/tests_cloaked/common/pi.gifand tanx is periodic over http://testonline.in/templates/default/images/tests_cloaked/common/pi.gif.  
**NOTE:**   
(a) Inverse of a periodic function does not exist.  
(b) Every constant function is always periodic, with no fundamental period.  
(c) If f(x) has a period T and g(x) also has a period T then it does not mean that   
f(x) + g(x) must have a period T.   
E.g : http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0004.gif  
(d) If f(x) has a period p, then http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0005.gifalso has a period p.  
(e) If f(x) has a period T then f(ax+b) has a period http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image008_0004.gif.  
  
**11. Composite Functions :**   
Let http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image010_0004.gifbe two functions. Then the function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image012_0003.gifdefined by http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014_0003.gifis called the composite of the two functions f & g.   
Diagramatically http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image016_0003.gif  
Thus the image of every http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image018_0000.gifunder the function gof is the g-image of the f-image of x.   
**Note that** gof is defined only if http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image020_0000.gif, f(x) is an element of the domain of g so that we can take its g-image. Hence for the product gof of two functions f and g, the range of f must be a subset of the domain of g.  
**Properties of Composite Functions:**   
(i) The composite of functions is not commutative i.e http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image022_0000.gif  
(ii) The composite of functions is associative i.e, if f, g, h are three functions such that fo(goh) and (fog)oh are defined, then fo(goh) = (fog)oh.  
(iii) The composite of two bijections is a bijection i.e if f and g are two bijections such that gof is defined, then gof is also a bijection.  
  
**12**. **Inverse of A Functions :**   
Let http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0008.gifbe a one-one and onto function, then their exists a unique function http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0005.gifsuch that http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image006_0006.gifThen g is said to be inverse of f. Thus http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image008_0005.gif.  
**Properties of Inverse Functions :**   
(i) The inverse of a bijection is unique   
(ii) If http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image002_0009.gifis a bijection and http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image004_0006.gifis the iverse of f, then http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image012_0004.gif, where IAand IBare identity functions on the sets A and B respectively.   
**Note that** the graphs of f and g are the mirror images of each other in the line y=x.  
(iii) The inverse of a bijection is also a bijection.  
(iv) If f and g are two bijections f: A http://testonline.in/templates/default/images/tests_cloaked/common/arrow-right.gifB; g :B http://testonline.in/templates/default/images/tests_cloaked/common/arrow-right.gifC then the inverse of gof exists and http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image014_0004.gifis also a bijection  
(v) Inverese of an even function is not defined.  
  
**13.** **Equal or Identical Functions :**  
Two functions f and g are said to be equal if :  
(i) The domain of f = the domain of g  
(ii) The range of f= the range of g and   
(iii) f(x) = g(x), for every x belonging to their common domain.   
E.g. http://testonline.in/templates/default/images/tests_cloaked/GOIIT/ch0029/paper_clip_image016_0004.gifare identical functions.  
  
**14. General :**   
If x, y are independent variables, then   
(i) f(xy) = f(x) + f(y) http://testonline.in/templates/default/images/tests_cloaked/common/double_arrow.giff(x) = k logx or f(x) = 0  
(ii) f(xy) = f(x). f(y) http://testonline.in/templates/default/images/tests_cloaked/common/double_arrow.giff(x) = xn, n R   
(iii) f(x+y) =f(x). f(y) http://testonline.in/templates/default/images/tests_cloaked/common/double_arrow.giff(x) = akx.   
(iv) f(x+y) = f(x)-f(y) http://testonline.in/templates/default/images/tests_cloaked/common/double_arrow.giff(x)=kx, where k is constant.